Understanding and Teaching Ratios and Proportional Relationships

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Goals for the presentation today on ratio and proportional relationships

An opportunity to think together about:

- Motivating the concept of ratio and using ratio language;
- Reasoning about ratio tables, double number lines, and strip diagrams to solve problems and develop understanding of proportional relationships;
- Distinguishing ratios from fractions but connecting ratios to fractions via unit rates;
- Using unit rates to solve problems;
- Examine graphs and equations for proportional relationships.

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IES Practice Guide on Fractions

Developing Effective Fractions Instruction for Kindergarten Through 8th Grade

Recommendation 4:
"Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems."

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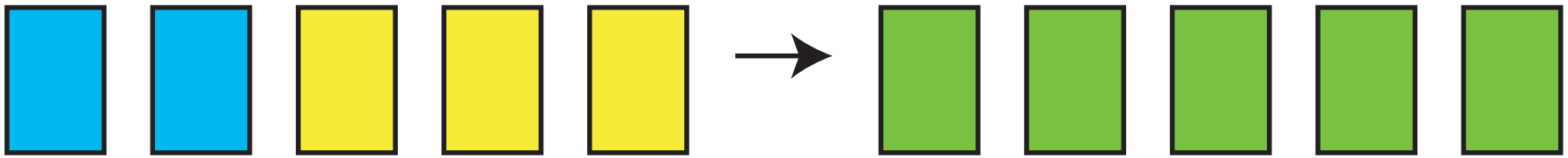
Common Core State Standards

Ratios and Proportional Relationships, Grades 6, 7

6.RP.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Motivating the concept of ratio

What happens when we mix 2 cups blue paint with 3 cups yellow paint?



That was 1 batch. What if we make more batches?

# of batches	1	2	3	4	5	6	7
# cups blue paint	2	4	6	8	10	12	14
# cups yellow paint	3	6	9	12	15	18	21
# cups green paint produced	5	10	15	20	25	30	35

What do these paint mixtures have in common?

Same shade of green. For every 2 cups blue, there are 3 cups yellow.

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Motivating the concept of ratio

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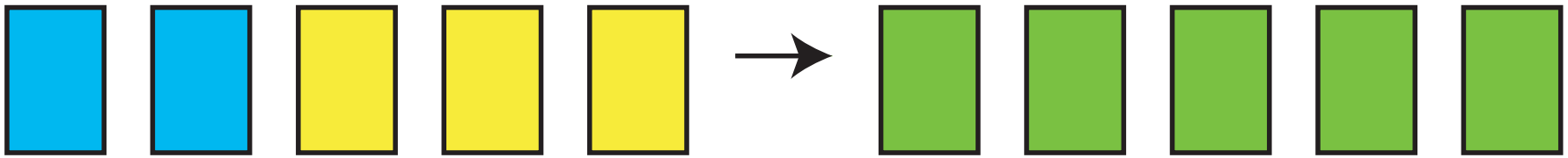

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What do these paint mixtures have in common?

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CCSS Ratios and Proportional Relationships

6.RP.1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

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A ratio table

Blue and yellow paint are mixed in the ratio 2 cups : 3 cups.
This means: **for every** 2 cups blue paint present, there are 3 cups yellow paint present.

In each mixture, blue and yellow paint are in a ratio of 2 to 3:

# of batches	1	2	3	4	5	6	7
# cups blue paint	2	4	6	8	10	12	14
# cups yellow paint	3	6	9	12	15	18	21
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Reasoning about ratio tables to solve problems

Blue and yellow paint are mixed in a ratio of 2 to 3 to make Green Goblin paint.
How many cups of blue paint and how many cups of yellow paint will you need to make 30 cups of Green Goblin paint?

# of batches	1	2	3	4	5	**6**	7
# cups blue paint	2	4	6	8	10	**12**	14
# cups yellow paint	3	6	9	12	15	**18**	21
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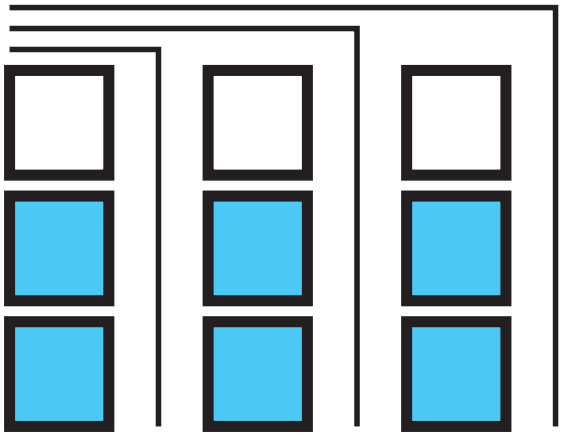
How are ratios and fractions different?

Equivalent ratios

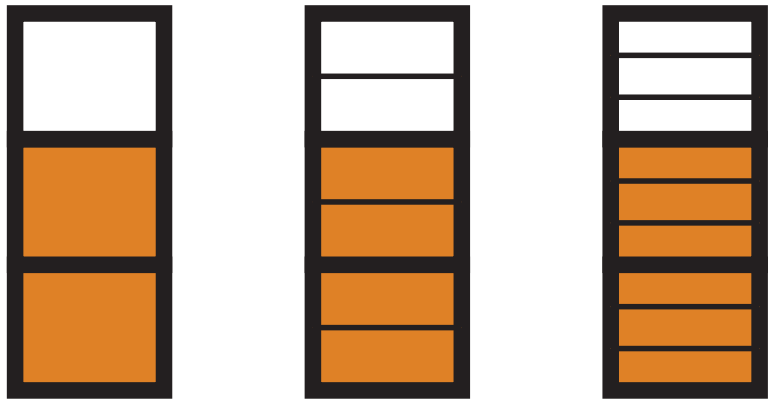


cups blue	2	4	6
total cups	3	6	9

more parts, same size parts

more total paint
more blue pigment

Equivalent fractions



$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$$

more parts, smaller parts

same whole amount
same portion

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Comparing mixtures

Abby's orange paint is made by mixing red and yellow paint in the ratio 1 cup : 3 cups.
Zack's orange paint is made by mixing red and yellow paint in the ratio 3 cups : 5 cups.
Are the two shades of orange the same? Why or why not?

What's a common student misconception?

Students sometimes think the paints are the same shade because each mixture has 2 more cups yellow than red or because you get Zack's paint by adding 2 cups red, 2 cups yellow to Abby's.

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Using ratio tables to compare mixtures

Abby's orange paint is made by mixing 1 cup red paint with 3 cups yellow paint.
Zack's orange paint is made by mixing 3 cups red paint with 5 cups yellow paint.

1. Make a ratio table for Abby's paint. Why do all the mixtures in the table have the same shade of orange?
2. Make a ratio table for Zack's paint. Why do all the mixtures in the table have the same shade of orange?
3. Look for common entries to compare the two mixtures.

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Using ratio tables to compare mixtures

Same amount of red.
Abby's has more yellow,
so Abby's is yellower,
Zack's is redder.

Abby's

cups red	cups yellow
1	3
2	6
3	9
4	12
5	15

Zack's

cups red	cups yellow
3	5
6	10
9	15
12	20
15	25

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Using ratio tables to compare mixtures

Abby's

cups red	cups yellow
1	3
2	6
3	9
4	12
5	15

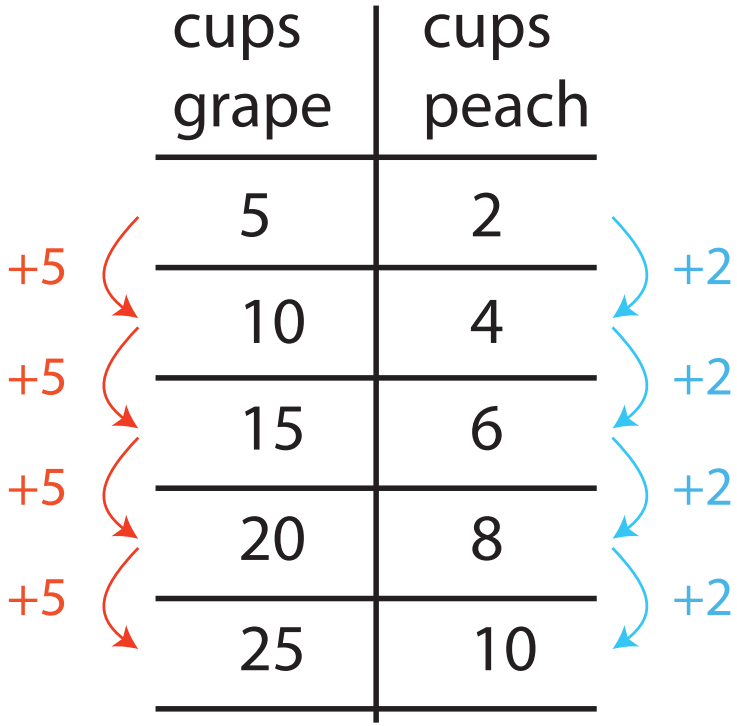
Zack's

cups red	cups yellow
3	5
6	10
9	15
12	20
15	25

Same amount of yellow.
Zack's has more red.
So Zack's is redder,
Abby's is yellower.

Additive structure in a ratio table

5 cups grape juice for every 2 cups peach juice



Can you see another structure?

Find unknown entries in a ratio table

5 cups grape juice for every 2 cups peach juice.
How can we find the unknown entries?

cups grape	cups peach
5	2
	16
100	
150	
	100

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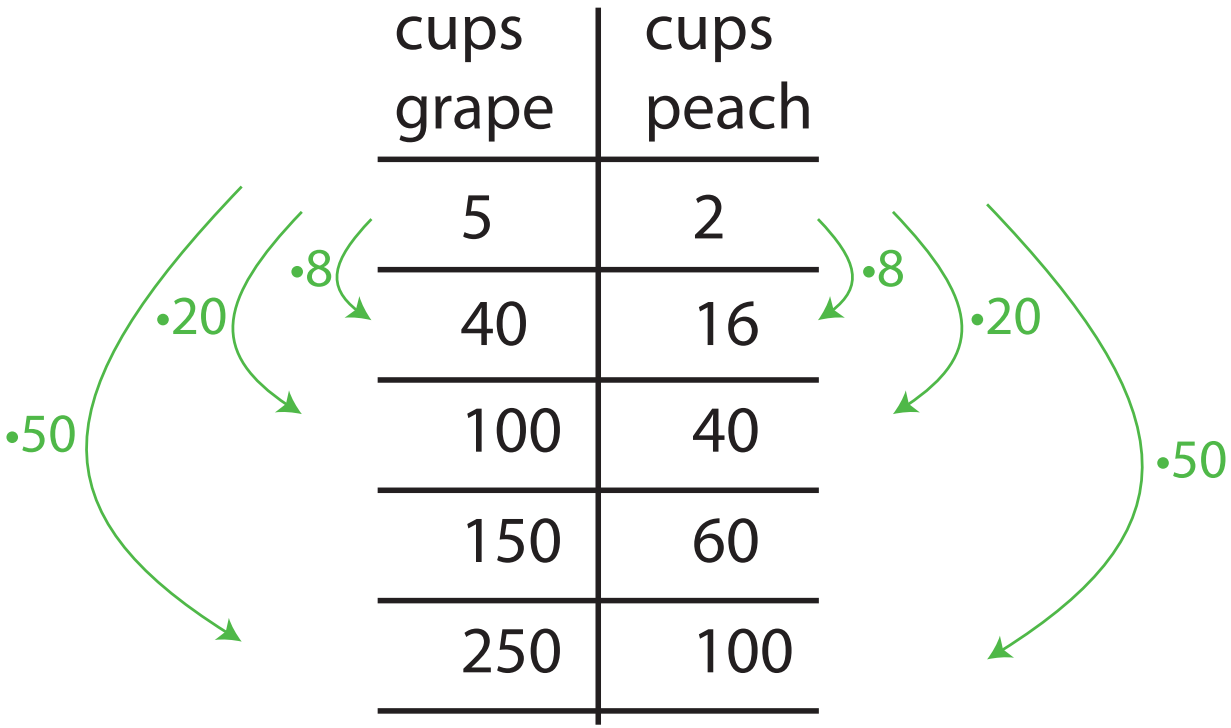
Multiplicative structure in a ratio table

5 cups grape juice for every 2 cups peach juice.



Unit rates

5 cups grape juice for every 2 cups peach juice.

cups grape	cups peach
5	2
	1
1	

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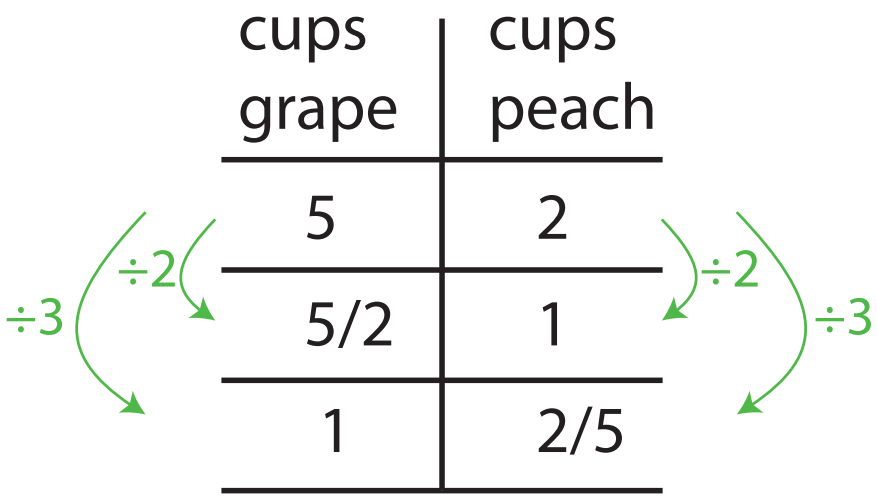

Unit rates:
5/2 cups grape juice for every **1** cup peach juice;
2/5 cups peach juice for every **1** cup grape juice.

Reasoning with unit rates

5 cups grape juice for every 2 cups peach juice.

1. How much grape juice should you use for 5 cups peach juice?
2. How much peach juice should you use for 3 cups grape juice?

cups grape	cups peach
5	2
5/2	1
	5
1	2/5
3	

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Reasoning with unit rates

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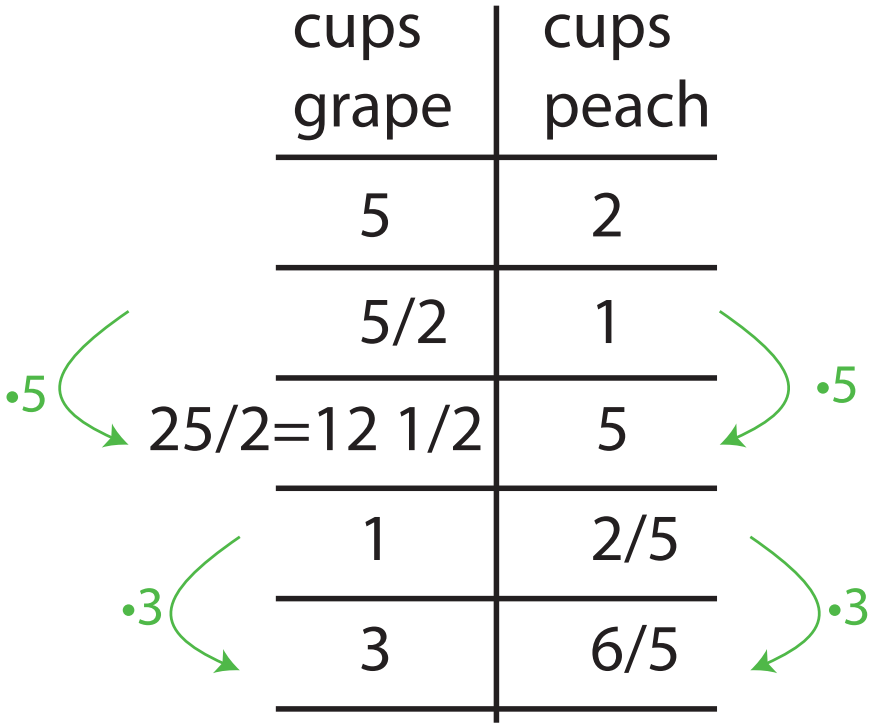

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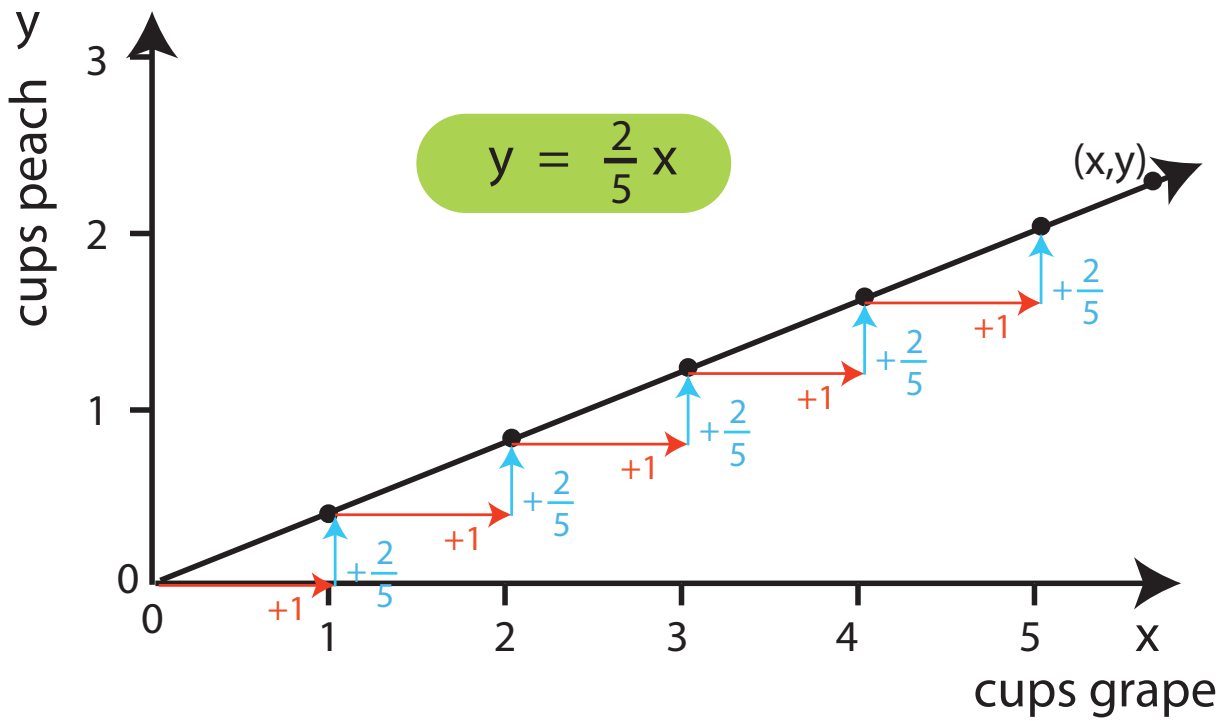
Showing a proportional relationship in a table, graph, equation

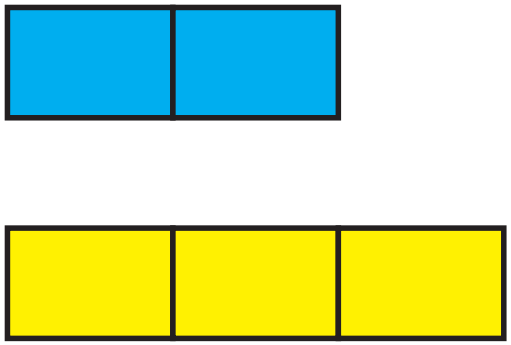
For every 5 cups grape juice, mix in 2 cups peach juice

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$



Reasoning with strip diagrams

Blue and yellow paint are mixed in a ratio of 2 to 3 to make green paint. How many pails of blue paint and how many pails of yellow paint will you need to make 30 pails of green paint?



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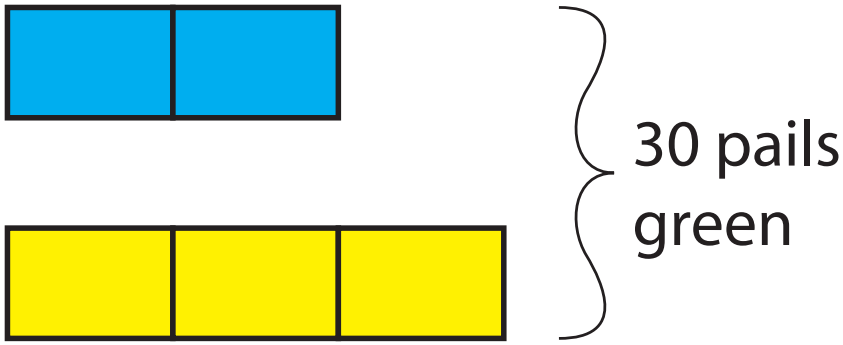

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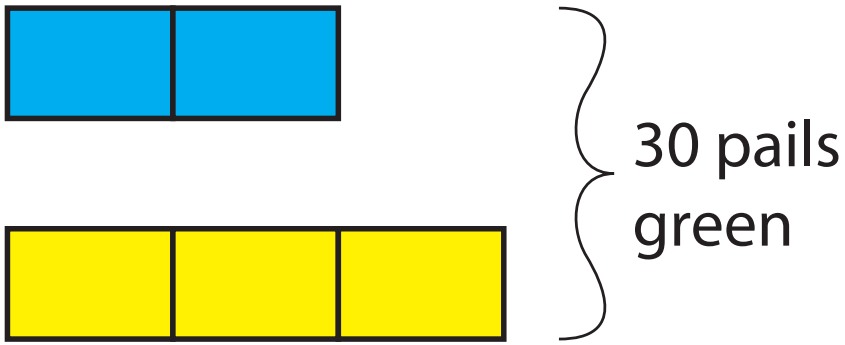

5 equal parts make 30 pails

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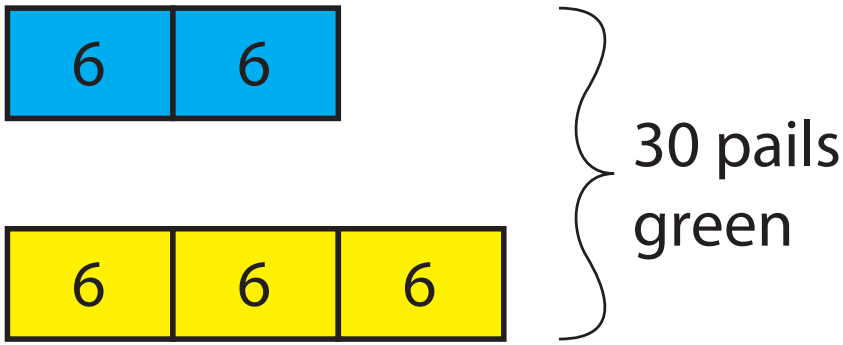

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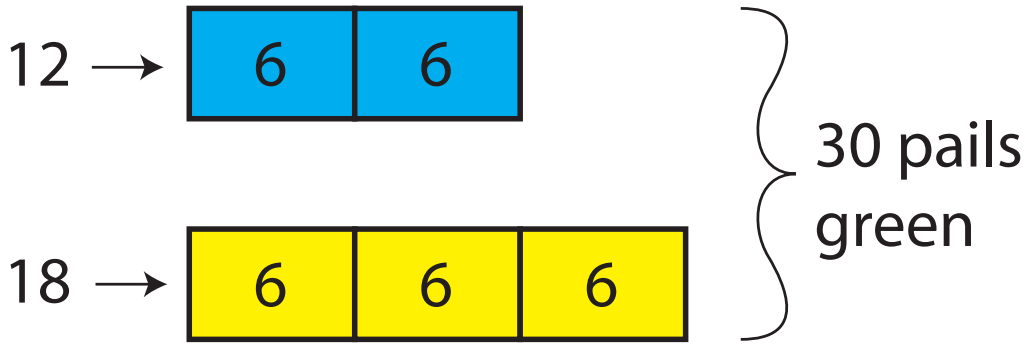
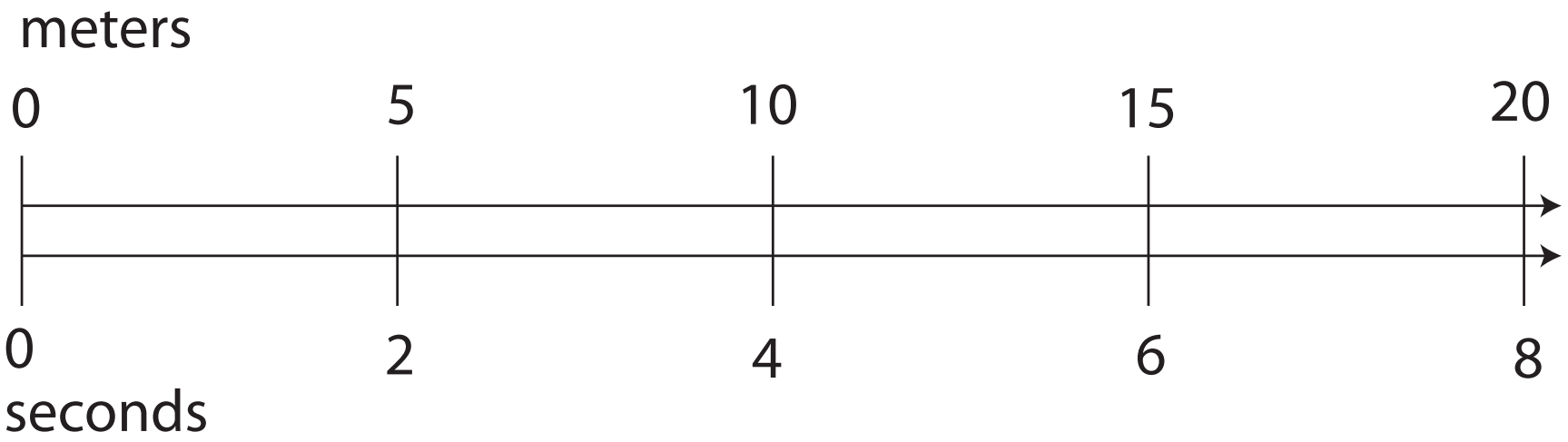


5 equal parts make 30 pails

Reasoning with double number lines

Asha runs 5 meters every 2 seconds.



Give some problems you can solve using this double number line.

How can you use the double number line to help you solve these problems:
How far does Asha run after 3 seconds? After 5 seconds?

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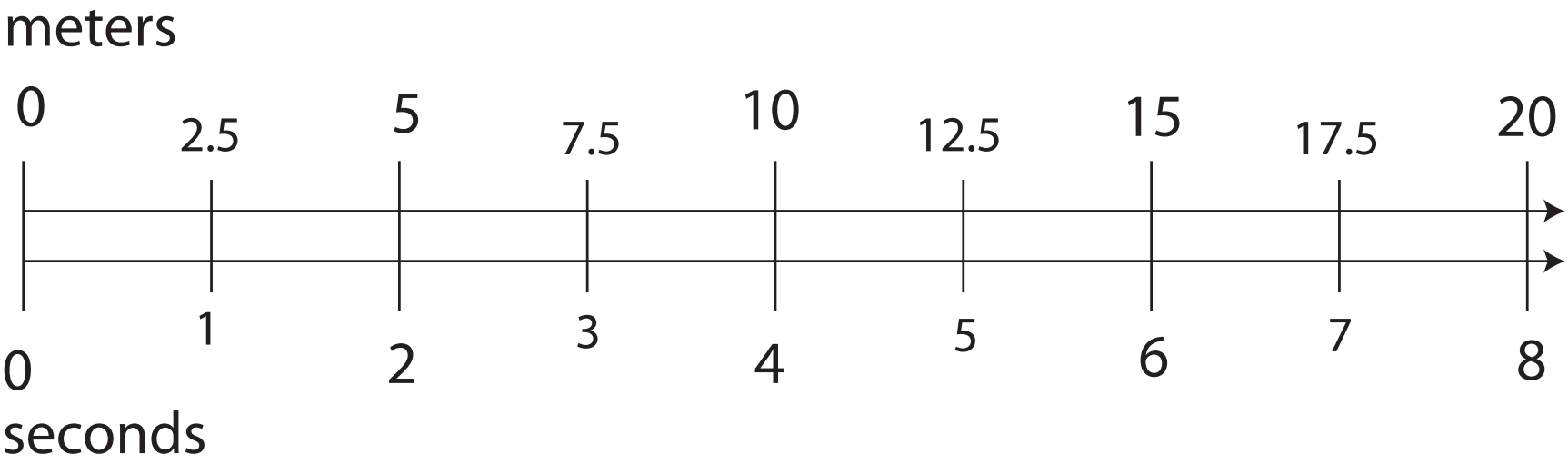
Reasoning with double number lines

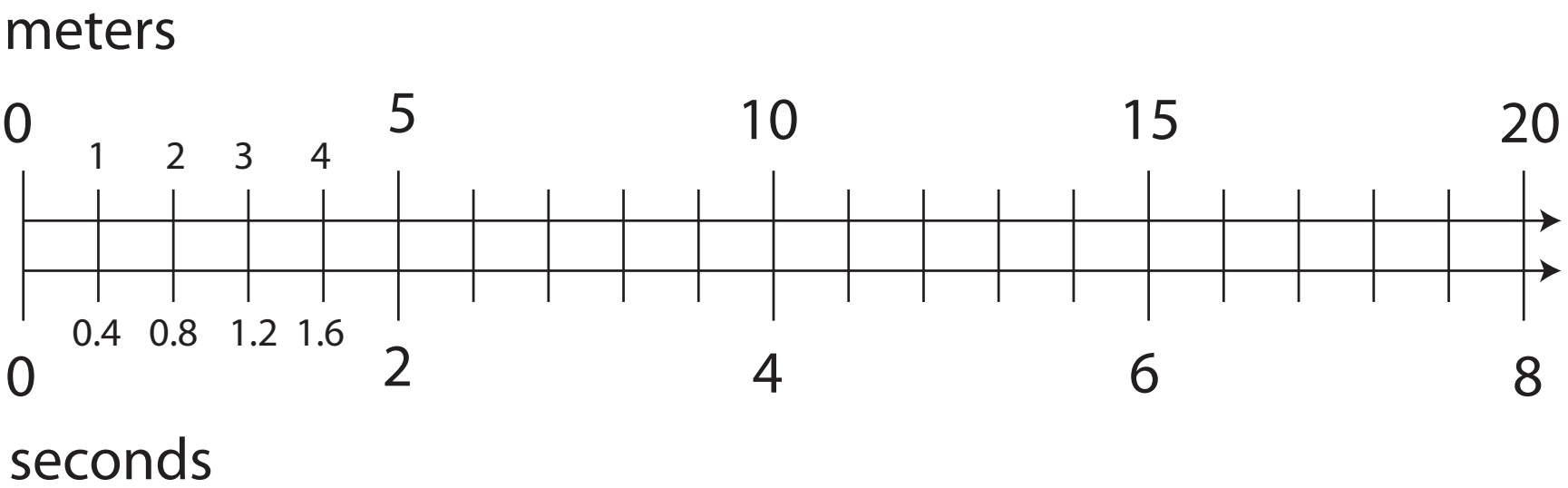
Asha runs 5 meters every 2 seconds.



Make a new double number line and mark it to help you solve these problems:
How long does it take Asha to run 3 meters? 1 meter?

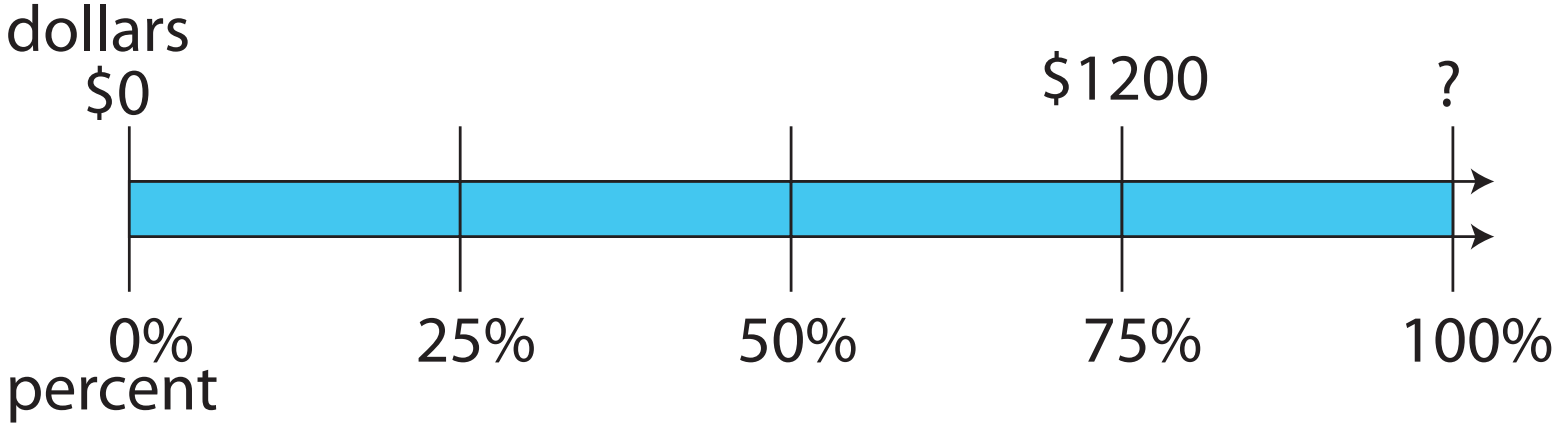
Reasoning with double number lines

Asha runs 5 meters every 2 seconds.



Reasoning about percent with double number lines

If 75% of the budget is $1200, then what is the full budget?



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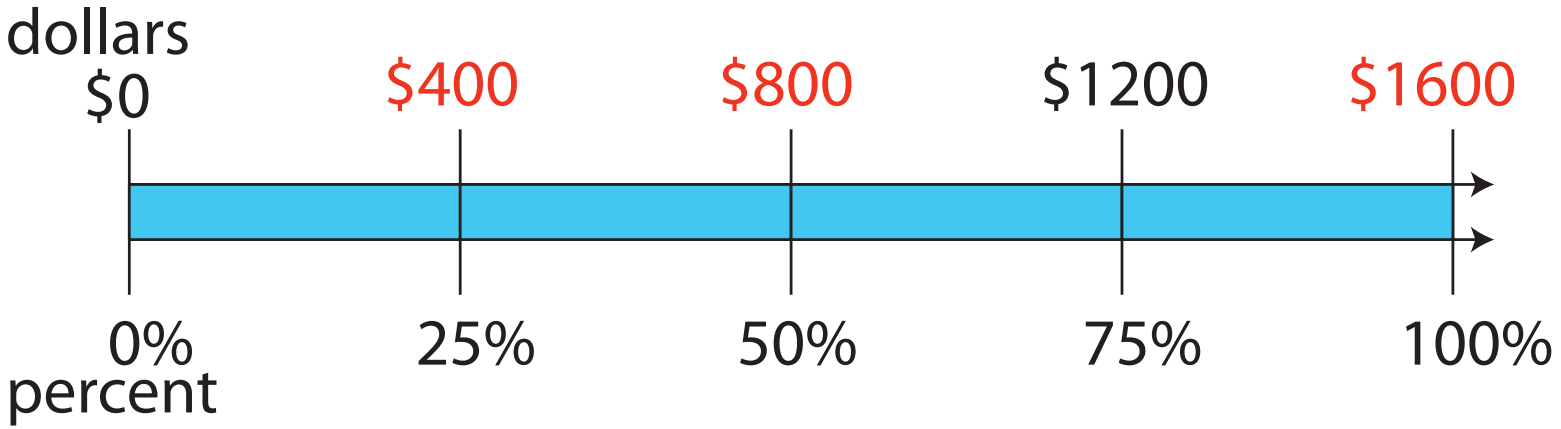

Thank you!

Questions? Comments?

Why are the following two problems *not* solved the same way?

Problem 1: After a 20% discount, a bike costs $160. How much did the bike cost before?

Problem 2: A bike costs $160 now, but its price will go up by 20%. What will it cost then?

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